

NAME: _____

LESSONS 17 AND 18 - SOLIDS OF REVOLUTION - SHELLS
MATH 16020

III Washers vs. Shells

So far, we have used disks and washers to find volumes. A washer is like a washer that you would see in a hardware store. It is very short, but the difference between the two radii can be significant.



Today we are going to use **shells** to find volumes. Shells are like a soup can that has no top or bottom. They can be tall, but the difference between the two radii is quite small.

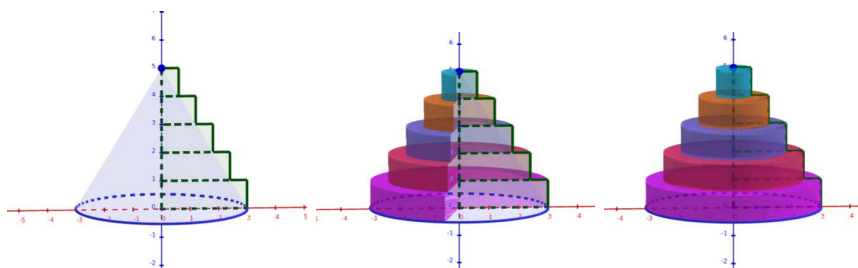


Whether we are working with washers or shells, it is important to remember that:

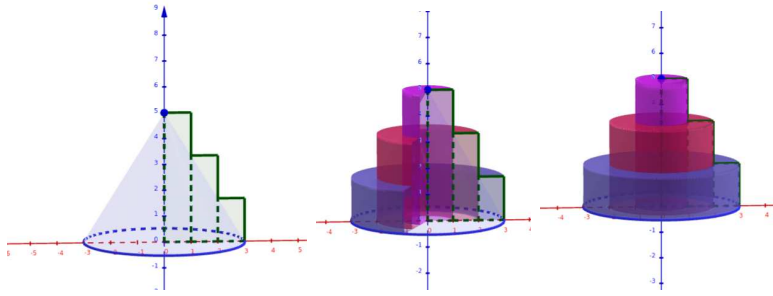
- The **centers** of the circles are at the **axis of rotation**.
- The **radii** of the circles extend **out from the axis of rotation**.

Let's take a look at the cone that we created in a previous class by rotating the triangle with vertices $(0, 0)$, $(3, 0)$, and $(0, 5)$ about the y -axis.

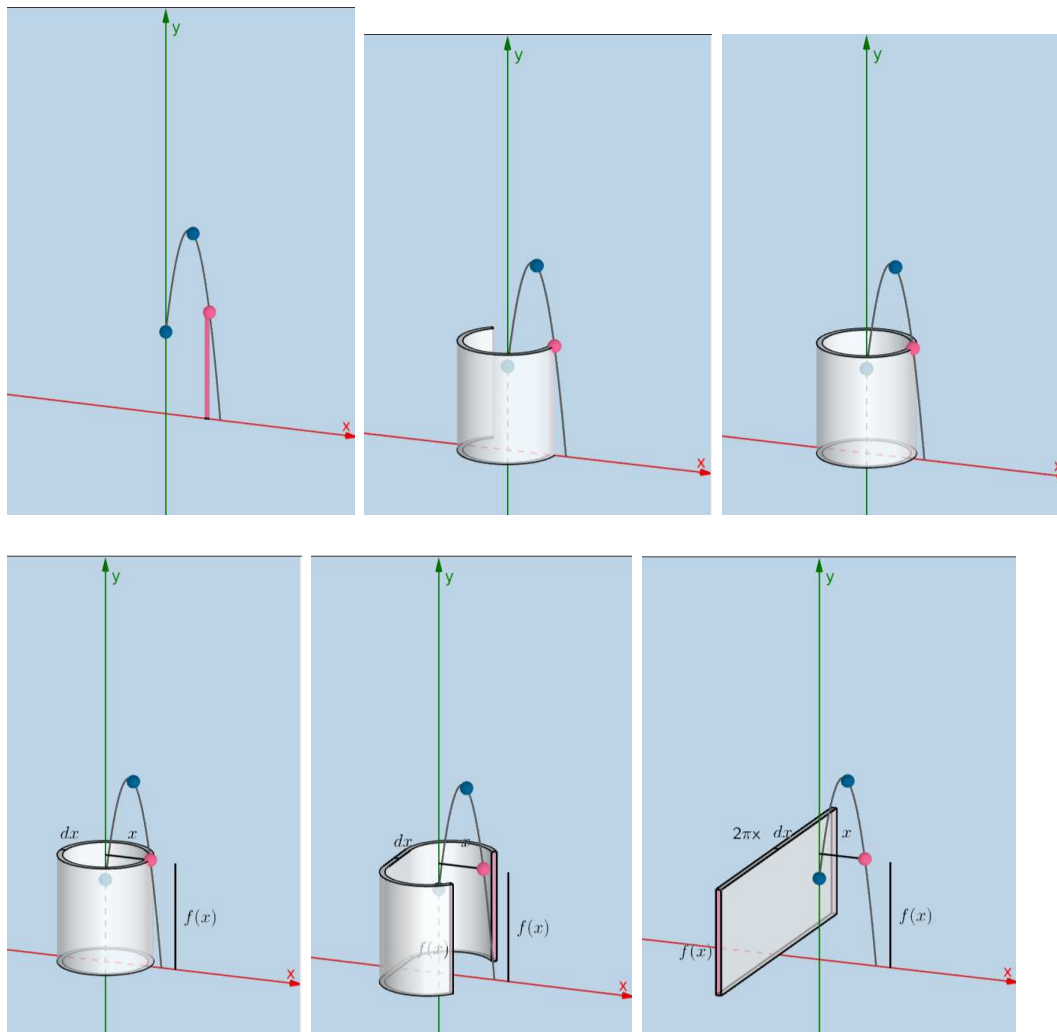
Pictures from Geogebra using washers:



Pictures from Geogebra using shells:



When we found volume using washers, we integrated the area of a circle with a hole punched out of the middle. When we find the volume using shells, we integrate the area of a rectangle. The following pictures come from Geogebra and were created by Tim Brzezinski.



What do we integrate?

- washers - $\pi(R^2 - r^2)$
- shells - $2\pi rh$

What variable do we use to integrate?

Axis of Rotation	x -axis or horizontal axis	y -axis or vertical axis
washers	dx	dy
shells	dy	dx

IV Examples

Example 1. Use shells to find the volume of the solid obtained by rotating the triangle with vertices $(0, 0)$, $(3, 0)$, and $(0, 5)$ about the y -axis.

Example 2. Find the volume of the solid obtained by revolving the region enclosed by the curves

$$y = 2x^5, \quad y = 0, \quad \text{and} \quad x = 1$$

about the y -axis.

Example 3. Find the volume of the solid obtained by revolving the region enclosed by the curves

$$x = 0 \quad \text{and} \quad x = 4y - y^2$$

about the x -axis.

Example 4. Find the volume of the solid obtained by revolving the region enclosed by the curves

$$x = 3 - y \quad \text{and} \quad x = 3 + 2y - y^2$$

about the x -axis.

Example 5. Find the volume of the solid obtained by revolving the region enclosed in the first quadrant by the curves

$$y = x^3 \quad \text{and} \quad y = 25x$$

about the y -axis.

Example 6. Set up but **DO NOT EVALUATE** an integral expression for finding the volume of the solid obtained by revolving the region enclosed by the curves

$$y = \sin(x), \quad x = 0, \quad x = \pi, \quad \text{and} \quad y = 0$$

about . . .

(a) the line $x = 2\pi$

(b) the line $y = 1$

Example 7. Set up but **DO NOT EVALUATE** an integral expression for finding the volume of the solid obtained by revolving the region the triangle with vertices $(0, 0)$, $(6, 4)$, and $(6, 0)$ about . . .

(a) the line $y = 10$

(b) the line $y = -2$

(c) the line $x = 8$

(d) the line $x = -3$